Self Organising Maps (SOM)

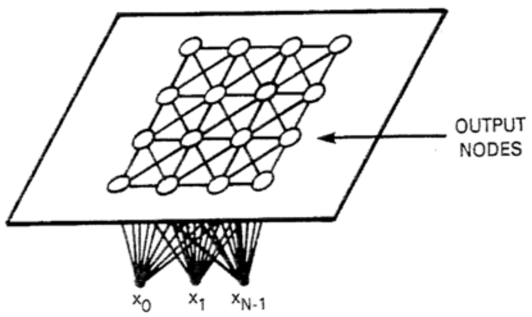
- Self organising is a process of unsupervised learning whereby significant patterns or features in the input data is discovered
- Leaning consists of adaptively modifying the weights of a network of locally interacting units in accordance with a learning rule until a useful configuration develops.

Self Organizing Maps

- Based on competitive learning(Unsupervised)
  - Only one output neuron activated at any one time
  - Winner-takes-all neuron or winning neuron
- In a Self-Organizing Map
  - Neurons placed at the nodes of a lattice
    - one or two dimensional
  - Neurons selectively tuned to input patterns
    - by a competitive learning process
  - Locations of neurons so tuned to be ordered
    - formation of topographic map of input patterns
  - ❖ Spatial locations of the neurons in the lattice -> intrinsic statistical features contained in the input patterns

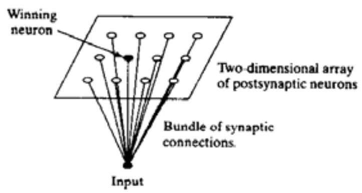
Self Organizing Maps

- Topology-preserving transformation



Basic Feature-mapping model

- Kohonen Model(1982)
  - Captures essential features of computational maps in Brain
    - remains computationally tractable



(b) Kohonen model

Formation Process of SOM

- After initialization for synaptic weights, three essential processes
  - Competition
    - Largest value of discriminant function selected
    - Winner of competition
  - Cooperation
    - Spatial neighbors of winning neuron is selected
  - Synaptic adaptation
    - Excited neurons adjust synaptic weights

Competitive Process

- Input vector, synaptic weight vector

  $\boldsymbol{x} = [x_1, x_2, \ldots, x_m]^T$

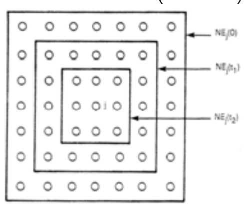
  $\boldsymbol{w}_j = [w_{j1}, w_{j2}, \ldots, w_{jm}]^T, j = 1, 2, 3, l$ *(initial weight of a node)*
- Best matching, winning neuron

  $i(\boldsymbol{x}) = \arg\min ||\boldsymbol{x}-\boldsymbol{w}_j||, j = 1,2,3,..,l$
- Determines the location where the topological neighborhood of excited neuron is centered

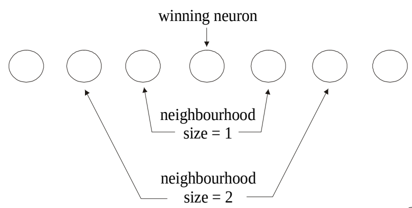
Cooperative Process

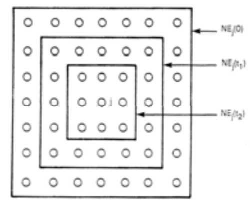
- For a winning neuron, the neurons in its immediate neighborhood excite more than those farther away
- topological neighborhood decay smoothly with lateral distance
  - Symmetric about maximum point defined by $d_{ij} = 0$
  - Monotonically decreasing to zero for $d_{ij} \rightarrow \infty$
  - Neighborhood function: Gaussian case $h_{j,i(x)} = \exp\left(-\frac{d_{j,i}^2}{2\sigma^2}\right)$
- Size of neighborhood shrinks with time

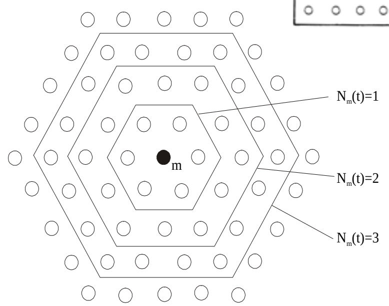
  $\sigma(n) = \sigma_0 \exp\left(-\frac{n}{\tau_1}\right), \quad n = 0,1,2,3$



Neighborhood







Pl ignore the notations
as the figures are taken from
various sources

Adaptive process

- Synaptic weight vector is changed in relation with input vector

  $\mathbf{w}_j(n+1) = \mathbf{w}_j(n) + \eta(n)\, h_{j,i(x)}(n)\, (\mathbf{x} - \mathbf{w}_j(n))$
- Applied to all neurons inside the neighborhood of winning neuron i
- Upon repeated presentation of the training data, weight tend to follow the distribution
- Learning rate $\eta(n)$: decay with time
- May decompose two phases
  - Self-organizing or ordering phase : topological ordering of the weight vectors
  - Convergence phase : after ordering, for accurate statistical quantification of the input space

Summary of SOM(1)

- Continuous input space of activation patterns that are generated in accordance with a certain probability distribution
- Topology of the network in the form of a lattice of neurons, which defines a discrete output space
- Time-varying neighborhood function defined around winning neuron.
- Learning rate decrease gradually with time, but never go to zero

Summary of SOM(2)

- Learning Algorithm
  1. Initialize w's
  2. Find nearest cell

     $i(\underline{x}) = \text{argmin}_j \| \underline{x}(n) - \underline{w}_j(n) \|$
  3. Update weights of neighbors

     $\underline{w}_j(n+1) = \underline{w}_j(n) + \eta(n)\, h_{j,i(x)}(n)\, [\, \underline{x}(n) - \underline{w}_j(n)\,]$
  4. Reduce neighbors and η
  5. Go to 2

SOM Examples